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LETTER TO THE EDITOR

Thermodynamics and cosmology

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Abstract. It is shown that Schücking and Spiegel's 'vivacity' can indeed be written as a combination of 1 and p^μ, in accordance with Chernikov's theorem.

Schücking and Spiegel (1970) have commented on the problem of the expansion of a relativistic gas. They proposed a kind of 'equilibrium' compatible with adiabatic expansion, i.e. a solution of the relativistic Boltzmann equation in a Robertson–Walker universe, such that the gas has zero bulk viscosity. In fact they found a quantity (the 'vivacity') defined by

$$I \equiv ER - m\dot{R}\tau \tag{1}$$

which in a particular situation ($\dot{R}$ = constant) is conserved both during collisions and along the world line of the particle between collisions. At first sight this seems very disturbing since it is known that 1 and p^μ form a basis in the space of additive collision invariants (Chernikov's theorem). Given this theorem, it has been shown that a relativistic gas composed of particles with non-zero rest mass cannot be in equilibrium in an expanding Robertson–Walker universe.

In their response, Stewart *et al* (1970) explain clearly why an expanding relativistic gas must in general produce entropy, but they avoid the vivacity problem by saying that such a quantity 'would call for the inclusion of the proper times of all the particles as statistical variables. In this case kinetic theory would not be applicable and so the result that an ideal gas cannot expand adiabatically need not hold'.

In this letter we shall explicitly calculate (1) in terms of the coordinates on the phase space of particles with rest mass m, and show that the vivacity is, indeed, a simple combination of 1 and p^μ. It follows that Robertson–Walker universes with R constant cannot admit an equilibrium adiabatic expansion of a relativistic gas.

In order to consider the vivacity, we start with the metric of Robertson–Walker universes

$$ds^2 \equiv d\tau^2 = dt^2 - R^2K^2\delta_{ij}\, dx^i\, dx^j \qquad i, j = 1, 2, 3 \tag{2}$$

where $K^2 = (1+\frac{1}{4}\kappa r^2)^{-2}$, $\kappa = -1, 0, 1$ and R is the expansion function; we shall take $\dot{R} = dR/dt$ = constant. (Units are such that $c = 1$.)

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Introducing the quantities

$$u^\alpha = \frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau} \qquad \alpha = 0, 1, 2, 3 \qquad (x^0 \equiv t)$$

$$v^i = \frac{u^i}{u^0}; \qquad a^i = \dot{v}^i = \frac{\mathrm{d}v^i}{\mathrm{d}t}; \qquad v^2 = \delta_{ij} v^i v^j \qquad i, j = 1, 2, 3$$

we may write the geodesic equations as

$$a^i = -\tfrac{1}{2}\kappa K x^i v^2 + [\kappa K v^j x_j - 2(\dot{R}/R) + R\dot{R}K^2 v^2] v^i. \tag{3}$$

It is trivial to show that $u^\alpha \partial_\alpha I = 0$; that is, the vivacity is conserved along particle trajectories.

From (2) we get directly that

$$\dot{\tau} \equiv \frac{\mathrm{d}\tau}{\mathrm{d}t} = (1 - R^2 K^2 v^2)^{1/2} \tag{4}$$

and a further differentiation yields, with the help of (3):

$$\ddot{\tau} = R\dot{R}K^2 v^2 \dot{\tau} = \frac{\dot{R}\dot{\tau}}{R}(1 - \dot{\tau}^2) \tag{5}$$

which can equally well be obtained from the zeroth component of the geodesic equation. In fact, (5) can be integrated twice to give

$$\tau = C_1 + \left(C_2 + \frac{2R(0)}{\dot{R}} t + t^2 \right)^{1/2} \tag{6}$$

where $R = \dot{R}t + R(0)$ when $\dot{R}$ is constant and C_1, C_2 are constants of integration. The energy of a particle in the gas is $E = mu^0 = m/\dot{\tau}$ and from (6) this can be expressed as

$$E = m\dot{R}\frac{\tau - C_1}{R}.$$

Therefore the vivacity is given by

$$I \equiv ER - m\dot{R}\tau = -m\dot{R}C_1 = \text{constant}.$$

Thus the vivacity I trivially satisfies Chernikov's theorem, and the adiabatically expanding equilibrium of a relativistic gas suggested by Schücking and Spiegel is not possible.

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References

Schücking E L and Spiegel E A 1970 *Comments Astrophys. Space Sci.* **1** 121
Stewart J M, MacCallum M A H and Sciama D W 1970 *Comments Astrophys. Space Sci.* **1** 206